hep-th/0208140

Vacuum Structure and Global Strings with Conical Singularities

Yoonbai Kim[1], O-Kab Kwon[2], Jongsu Sohn[3]

BK21 Physics Research Division and Institute of Basic Science, Sungkyunkwan University, Suwon 440-746, Korea

Abstract

Vacuum structure and global cosmic strings are analyzed in the effective theory of self-interacting O(2) scalar fields on (3+1)-manifolds with conical singularities. In the context of one-loop effective action computed by heat-kernel methods with ζ-function regularization, we find an inhomogeneous vacuum of minimum energy and suggest some reason why low-energy global strings are likely to be generated at the conical singularities.

Keywords: Cosmic strings; Conic singularity; Effective potential

[1]yoonbai@skku.ac.kr
[2]okwon@newton.skku.ac.kr
[3]jongsu@newton.skku.ac.kr

1 Introduction

Cosmic strings remain somewhere in our universe as relics of cosmological phase transition at early times. These linear topological defects have been extensively studied in connection with cosmological problems [1]. Global strings were proposed as a candidate of producing primordial density perturbations leading to galaxy formation [2]. However, the original purpose seems unlikely to be achieved due to radiation of Goldstone bosons around oscillating closed loops of strings [3] and other stringent astrophysical bounds [4]. Recently other physical applications are also investigated, *e.g.*, time evolution of global string network [5] and possible generation of several types of global strings in high density QCD [6].

When infinite straight U(1) global strings are coupled to Einstein gravity, static solutions with cylindrical symmetry encounter unavoidable singularity [7]. With a negative cosmological constant, global vortices become seeds for charged BTZ black holes in (2+1) dimensions without physical singularity [8]. Such spacetime structure with horizons is extended to a viable model of 6-dimensional Randall-Sundrum type brane worlds, which is composed of our world on the 3-brane and 2 extra-dimensions identified with spatial plane of global vortices [9].

The questions we are interested in in this paper are the vacuum structure and the global strings in the effective theory with conical singularities. Suppose that an infinite straight local cosmic string was generated throughout a cosmological phase transition in the very early universe, *e.g.*, a Nielsen-Olesen vortex string at grand unification (GUT) scale. If there has been no other phase transition up to the late-time transition of a very low energy, *e.g.*, electroweak (EW) scale or lower scale, the supermassive local cosmic string may be observed through some gravitational effects, i.e., background spacetime has a deficit angle. Even though a phase transition occurs at the low energy, released latent heat or other fluctuations may be not enough to *melt* this high-energy cosmic string. So the survived string may affect the new vacuum structure of spontaneous symmetry breaking. In section 2, we assume an O(2) scalar theory with conical singularity in the background spacetime for tractability, and compute one-loop effective action including a δ-function singularity. In section 3, we analyze vacuum of the effective theory and show that an inhomogeneous vacuum, jumping from symmetric local maximum at the points of conical singularities to broken global minimum at the other space, is minimum energy configuration instead of homogeneous broken vacuum. In section 4, we address our main issue that the production of a global string along the supermassive local cosmic string seems favorable and natural.

The obtained low-energy global string containing the supermassive local cosmic string core looks like an infinite straight candle with a heavy wick. This kind of possibilities was firstly proposed in the case of superconducting strings, *e.g.*, the O(10) string with an inner core of $\tilde{\mathrm{U}}(1)$ magnetic flux [10]. It may also be understood as a simplified field-theoretic calculation of daily life experience that the structure of extend objects can easily formed at the site of a defect or dust when a vapor (or a liquid) is cooled down to the liquid (or the solid).

2 Effective action in the presence of a cosmic string

Suppose that a straight cosmic string was produced throughout a cosmological phase transition in the very early universe. When its species is a local cosmic string, *e.g.*, a string-like object of a Nielsen-Olesen vortex of Abelian Higgs model, the energy density is localized around the stringy core of the cosmic string, of which size is the inverse of the Higgs mass. At an energy scale much lower than that of the Higgs mass, the cosmic string looks like an extremely-thin straight wire so that its energy-momentum tensor can be approximated as a δ-function on a plane orthogonal to the string direction :

$$T^{\mu}_{\ \nu} \sim M_0^2 \mathrm{diag}(1,0,0,1)\delta(x)\delta(y), \tag{2.1}$$

where string direction is chosen by z-axis and mass of the string per unit length, M_0^2, is roughly of order the square of the scale of the phase transition.

The existence of such massive object in the early universe is likely to be detected by gravitational effect. Spacetime structure of the cosmic string (2.1) is determined by Einstein equations and then, in cylindrical coordinates, metric is expressed as follows

$$ds^2 = -dt^2 + dr^2 + r^2(1-4GM_0^2)^2 d\theta^2 + dz^2. \tag{2.2}$$

This spacetime is flat everywhere except for the site of the string core at $r=0$, and the effect of the local cosmic string is detected only by a deficit angle $2\pi - \gamma = 8\pi G M_0^2$ on the (x,y)-plane (or (r,θ)-plane) C_γ. Except for supermassive scale ($4GM_0^2 \geq 1$), C_γ describes nothing but a cone [11].

From now on let us consider a model at low energy in the presence of a very massive straight cosmic string. To be specific, we take into account massless O(2) linear sigma model in the gravitational background of a straight local cosmic string (2.2), described by the action

$$S = \int_{\mathcal{M}} d^4x \left(-\frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a - \frac{\lambda}{24}(\phi^2)^2 \right), \tag{2.3}$$

where $\phi^2 = \phi_a\phi_a$ ($a = 1,2$) and the base manifold is $\mathcal{M} = M^2 \times C_\gamma$. The situation with which we are dealing should involve two ingredients : One is a phase transition and the other some effect of high-energy cosmic string skeleton. In this paper, we will compute effective action $\Gamma_{\rm tot}(\phi)$ so that the vacuum shifts from symmetric vacuum $\langle\phi\rangle = 0$ to new broken vacuum $\langle\phi\rangle \neq 0$ and a few singular terms are induced in effective potential, due to δ-function singularity at the string site. It is actually enough to calculate the one-loop effective potential in order to fulfill the above demands up to leading order.

Here we compute the one-loop effective action $\Gamma^{(1)}$ by using background field methods near classical field ϕ_a^c. In the context of Euclidean path integral formalism, the one-loop contribution $\Gamma^{(1)}$ to the effective action is

$$\begin{aligned}\Gamma^{(1)}(\phi_c) &= -\ln\int \mathcal{D}\phi^q\, e^{-\frac{1}{2}\int_{\mathcal{M}_{\rm E}} d^4x\, \phi_a^q A_{ab}\phi_b^q} & (2.4)\\ &= \frac{1}{2}\ln\det\left(\frac{A}{\mu^2}\right). & (2.5)\end{aligned}$$

In the second line of Eq. (2.5) an arbitrary mass parameter μ was introduced to keep the effective action dimensionless. Physicswisely, it will play a role of an ultraviolet cutoff and will be identified as the scale of the phase transition. The specific form of the operator A after Euclideanization is

$$A_{ab} = -\left(\partial_t^2 + \partial_z^2 + \Delta_c - \frac{1}{6}\lambda(\phi^c)^2\right)\delta_{ab} + \frac{\lambda}{3}\phi_a^c\phi_b^c, \tag{2.6}$$

where under the metric (2.2) the Laplace-Beltrami operator Δ_c in Eq. (2.6) is expressed by

$$\Delta_c = \partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\theta^2. \tag{2.7}$$

It is well-known that ζ-function regularization is an effective scheme for the calculation of the one-loop effective action (2.5) [12]. In this method the ζ-function with eigenvalues λ_j's of the differential operator A takes the form

$$\begin{aligned}\zeta(s|A) &= \sum_j \lambda_j^{-s} = \frac{1}{\Gamma(s)}\sum_j\int_0^\infty d\tau\, \tau^{s-1}e^{-\lambda_j\tau}\\ &= \frac{1}{\Gamma(s)}\int_0^\infty d\tau\, \tau^{s-1}\int_{\mathcal{M}_{\rm E}} d^4x\, K(x,x,\tau), & (2.8)\end{aligned}$$

where the kernel $K(x,y,\tau)\equiv\langle x|e^{-\tau A}|y\rangle$ satisfies the equation

$$-\partial_\tau K(x,y,\tau) = AK(x,y,\tau), \tag{2.9}$$

and its boundary condition is given by a δ-function $\delta^4_{\mathcal{M}_{\rm E}}(x-y)$ on the manifold $\mathcal{M}_{\rm E} = R^2\times C_\gamma$

$$\lim_{\tau\to 0} K(x,y,\tau) = \delta^4_{\mathcal{M}_{\rm E}}(x-y). \tag{2.10}$$

Therefore the one-loop effective action (2.5) is expressed in terms of the ζ-function (2.8)

$$\Gamma^{(1)}(\phi_c) = -\frac{1}{2}\zeta'(0|A) - \frac{1}{2}\ln\mu^2\zeta(0|A), \tag{2.11}$$

where $\zeta'(0|A) = \frac{d\zeta(s|A)}{ds}|_{s=0}$.

Though the background spacetime is spatially inhomogeneous and static vortex configurations will be taken into account at the subsequent sections, we compute the one-loop effective potential in this section. This calculation is enough for the above requirement such as the phase transition and the effective terms due to conical singularities. Now let us turn off spacetime dependence of the fields ϕ_a, and denote the classical field ϕ_c as ϕ in what follows for simplicity. Then the ζ-function in the one-loop effective action (2.11) is simplified as follows

$$\begin{aligned}\zeta(s|A) &= \frac{1}{\Gamma(s)}\int_0^\infty d\tau\ \tau^{s-1}\int_{\mathcal{M}_{\rm E}} d^4x\ {\rm tr}\langle x|e^{-\tau\left[-(\partial_t^2+\partial_z^2+\Delta_c-\frac{\lambda}{6}\phi^2)\delta_{ab}+\frac{\lambda}{3}\phi_a\phi_b\right]}|x\rangle \\ &= \frac{V_2}{4\pi\Gamma(s)}\int_0^\infty d\tau\ \tau^{s-2}\int_{C_\gamma} d^2x\ \langle x|e^{\tau\Delta_c}|x\rangle\left(e^{-\frac{\tau\lambda}{6}\phi^2}+e^{-\frac{\tau\lambda}{2}\phi^2}\right), \qquad (2.12)\end{aligned}$$

where 'tr' denotes the trace over internal index a, b, and V_2 is 2-dimensional volume of the (t, z)-coordinates. Using a well-known result for the case with conical singularities [13]

$$\int_{C_\gamma} d^2x\ \langle x|e^{-\tau(-\Delta_c)}|x\rangle = \frac{V(C_\gamma)}{4\pi\tau}+\frac{1}{12}\left(\frac{2\pi}{\gamma}-\frac{\gamma}{2\pi}\right), \qquad (2.13)$$

we have

$$\begin{aligned}\zeta(s|A) &= \frac{V_2V(C_\gamma)}{16\pi^2(s-1)(s-2)}\left[\left(\frac{\lambda\phi^2}{6}\right)^{2-s}+\left(\frac{\lambda\phi^2}{2}\right)^{2-s}\right] \\ &\quad +\frac{V_2\gamma_0}{16\pi^2(s-1)}\left[\left(\frac{\lambda\phi^2}{6}\right)^{1-s}+\left(\frac{\lambda\phi^2}{2}\right)^{1-s}\right], \qquad (2.14)\end{aligned}$$

where $V(C_\gamma)$ is volume of the cone C_γ and $\gamma_0 = \frac{\pi}{3}\left(\frac{2\pi}{\gamma}-\frac{\gamma}{2\pi}\right)$. Therefore, substitution of Eq. (2.14) into Eq. (2.11) leads to the one-loop effective potential

$$\Gamma^{(1)}(\phi) = \frac{5V_2V(C_\gamma)}{1152\pi^2}\lambda^2\phi^4\left[\ln\left(\frac{\lambda\phi^2}{2\mu^2}\right)-\frac{1}{10}\ln 3-\frac{3}{2}\right]+\frac{V_2\gamma_0}{48\pi^2}\lambda\phi^2\left[1+\frac{1}{4}\ln 3-\ln\left(\frac{\lambda\phi^2}{2\mu^2}\right)\right]. \qquad (2.15)$$

We obtain an effective action by adding the classical action (2.3) and the one-loop effective potential (2.15), which coincides with that containing the leading one-loop quantum correction in the derivative expansion

$$\Gamma_{\rm tot}(\phi) = \int_{\mathcal{M}} d^4x\left\{-\frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - V_{\rm eff}(\phi)-\frac{\gamma_0\delta_c(x)}{48\pi^2}\lambda\phi^2\left[1+\frac{1}{4}\ln 3-\ln\left(\frac{\lambda\phi^2}{2\mu^2}\right)\right]\right\}, \qquad (2.16)$$

where $\delta_c(x)$ is δ-function on the cone C_γ and $V_{\rm eff}(\phi)$ is regular part of the effective potential :

$$V_{\rm eff}(\phi) = \frac{\lambda}{24}\phi^4+\frac{5\lambda^2}{1152\pi^2}\phi^4\left[\ln\left(\frac{\lambda\phi^2}{2\mu^2}\right)-\frac{1}{10}\ln 3-\frac{3}{2}\right]+\frac{5\mu^4}{192\cdot 3^{\frac{4}{5}}\pi^2}\exp\left(2-\frac{96\pi^2}{5\lambda}\right). \qquad (2.17)$$

In Eq. (2.17) we adjusted a cosmological constant to zero at its true minimum and subtracted the energy of the cosmic string since it does not change physics of our interest.

3 Inhomogeneous vacuum due to conical singularities

As well-known, the effective potential $V_{\rm eff}(\phi)$ in Eq. (2.17) comprises a local maximum at $\phi = 0$ with a positive cosmological constant and the true vacuum is attained at

$$\phi(\equiv v) = \sqrt{\frac{2\mu^2}{\lambda}}\ \exp\left[\frac{\lambda(10+\ln 3) - 96\pi^2}{20\lambda}\right]. \tag{3.18}$$

An intriguing observation should be noted. At such broken vacuum with vanishing cosmological constant, the energy is not zero even for the homogeneous vacuum configuration of $\langle\phi\rangle = v$ but includes additional positive contribution from the site of the cosmic string ($r = 0$), which is proportional to the length of the string. The energy per unit length along z-axis for the broken vacuum is

$$E_z\big|_{\phi=v} = \frac{\gamma_0(192\pi^2 + 3\lambda\ln 3)}{160\cdot 3^{\frac{9}{10}}\pi^2}\exp\left(1 - \frac{48\pi^2}{5\lambda}\right)\frac{\mu^2}{\lambda} > 0. \tag{3.19}$$

To minimize its energy, the vacuum configuration should be static and cylindrically-symmetric along the z-axis. Let us look into another possibility of inhomogeneous vacuum that the scalar field ϕ has r-dependence. Introducing dimensionless quantities

$$\phi(x) = \frac{\mu}{\sqrt{\lambda}} f(r) \qquad \left(\tilde{v} = \sqrt{2}\ \exp\left[\frac{\lambda(10+\ln 3) - 96\pi^2}{20\lambda}\right]\right), \tag{3.20}$$

$$r \to \tilde{r} = \mu r, \quad \delta_c(x) \to \tilde{\delta}(\tilde{x}) = \frac{1}{\mu^2}\delta_c(x), \tag{3.21}$$

we express the energy per unit length along the z-axis such as

$$E_z = \frac{\mu^2}{\lambda}\int_0^\infty d\tilde{r}\ \tilde{r}\int_0^\gamma d\theta\left[\frac{1}{2}\left(\frac{df}{d\tilde{r}}\right)^2 + \tilde{V}_{\rm eff}(f) + \tilde{\delta}(\tilde{x})\ U(f)\right], \tag{3.22}$$

where dimensionless part of the effective action (2.17) becomes

$$\tilde{V}_{\rm eff}(f) = \frac{f^4}{24} + \frac{5\lambda}{1152\pi^2}f^4\left(\ln\frac{f^2}{2} - \frac{1}{10}\ln 3 - \frac{3}{2}\right) + \frac{5\lambda}{192\cdot 3^{\frac{4}{5}}\pi^2}\exp\left[2\left(1 - \frac{48\pi^2}{5\lambda}\right)\right], \tag{3.23}$$

$$U(f) = \frac{\gamma_0\lambda}{48\pi^2}f^2\left(1 + \frac{1}{4}\ln 3 - \ln\frac{f^2}{2}\right). \tag{3.24}$$

From here on, we denote the dimensionless quantities without tilde notation for convenience.

Outside the core of the cosmic string ($r > 0$), the singular part of the effective potential, $\delta(r)U(f)/\gamma r$, does not contribute to the energy functional (3.22). Consider the one-parameter family of configurations,

$$f_\kappa(r) \equiv f(\kappa r). \tag{3.25}$$

If $f(r)$ be the vacuum at $r > 0$, then $f(r)$ is an extremum of the energy functional. Since the derivative term in Eq. (3.22) is invariant under the scaling, the extremum condition provides $V_{\rm eff}(f) = 0$ so that $f = v$ should be the vacuum at $r > 0$. As mentioned previously, extension of the vacuum $f = v$ to the origin has positive energy contribution from the singular potential like as (3.19). The inhomogeneous vacuum would be given by a stationary solution of equation of motion

$$\frac{1}{r}\frac{d}{dr}r\frac{df}{dr} - \frac{dV_{\rm eff}(f)}{df} = \frac{\delta(r)}{r}\frac{dU(f)}{df}. \tag{3.26}$$

Since second term at left-hand side of Eq. (3.26) is finite at the origin, integration of it from zero to an infinitesimal ε gives

$$\varepsilon \left.\frac{df(r)}{dr}\right|_{r=\varepsilon} = \left.\frac{dU(f(r))}{df}\right|_{r=0}. \tag{3.27}$$

Right-hand side of Eq. (3.27) is a finite constant so that its solutions are classified into two :

$$\lim_{\varepsilon\to 0} f(\varepsilon)\left\{\begin{array}{ll} \sim \ln\varepsilon & \text{when } \left.\frac{dU}{df}\right|_{r=0} \neq 0 \\ = 0 \text{ or } \sqrt[8]{48} & \text{when } \left.\frac{dU}{df}\right|_{r=0} = 0 \end{array}\right. . \tag{3.28}$$

For the singular solution proportional to $\ln\varepsilon$, the energy per unit length along the z-axis (3.22) is logarithmically divergent due to the derivative term in Eq. (3.22), i.e., $E_z \sim \lim_{\varepsilon\to 0}\int^{\varepsilon} dr\, r(d\ln r/dr)^2 + \cdots \sim \ln\varepsilon$ [14]. So this cannot be a vacuum solution. Note that $f(0) = \sqrt[8]{48}$ is also ruled out because the valid range of f should not be much larger than v. Therefore, the remaining possibility in Eq. (3.28) is $f(0) = 0$.[4] Since the δ-function term is also invariant under the scale transformation (3.25), the scaling behavior of the energy functional (3.22) tells us that the energetically favored configuration is

$$f(r) = \left\{\begin{array}{ll} 0 & \text{at } r = 0 \\ v & \text{at } r \neq 0 \end{array}\right. . \tag{3.29}$$

For this configuration, both the regular and singular terms of the effective potential in Eq. (3.22) have vanishing energy contribution. One may easily notice that the derivative term also does not contribute to the energy according to the following estimation

$$\int_0^{\varepsilon} dr\; r\,\frac{1}{2}\left(\frac{df}{dr}\right)^2 \sim \left(\varepsilon\left.\frac{df}{dr}\right|_{r=\varepsilon}\right)^2 = 0. \tag{3.30}$$

Now we find the true vacuum of minimum energy. When the phase transition was accomplished so that the broken vacuum is realized almost everywhere, the site of the supermassive cosmic string still remains in the symmetric phase. Amazingly enough,

[4]Though $f \to \infty$ looks like global minimum with negative infinite energy in Eq. (3.22), $f \gg \mu/\sqrt{\lambda}$ is unphysical because the valid range of f in the computation of effective potential (2.15) does not allow such limit.

such nonanalytic inhomogeneous vacuum has zero energy. Suppose that the core region of the symmetric phase swells and becomes connected smoothly to the broken vacuum at asymptotic region. Then, the radiation of massless Goldstone bosons is naturally initiated [3] and its final destination is expected to be the obtained nonanalytic vacuum (3.29).

4 Global string with massive cosmic string wick

In the previous section we have shown that the effective action obtained in section 2 favors the symmetry-broken vacuum $\langle\phi\rangle = v$ almost everywhere but does the symmetric vacuum $\langle\phi\rangle = 0$ at the site of the massive cosmic string wick. Since a representative topological defect of this O(2) scalar theory is global vortex string, let us study infinite global vortex-line along z-axis, which connects smoothly the symmetric vacuum at the vortex center and the broken vacuum at exterior region of its core.

Field configuration of the global string is static and cylindrically-symmetric along the z-axis in order to minimize its energy. For an appropriate ansatz to make the field single-valued

$$\phi_1 + i\phi_2 = \frac{\mu}{\sqrt{\lambda}}\, e^{i\bar{n}\theta} f(r), \tag{4.31}$$

the energy per unit length along the z-axis involves a topological term proportional to $\bar{n}^2$;

$$E_z = \frac{\mu^2}{\lambda}\gamma \int_0^\infty d\tilde{r}\;\tilde{r}\left[\frac{1}{2}\left(\frac{df}{d\tilde{r}}\right)^2 + \frac{\bar{n}^2}{2\tilde{r}^2}f^2 + \tilde{V}_{\rm eff}(f) + \frac{\gamma_0}{48\pi^2\gamma}\frac{\delta(\tilde{x})}{\tilde{r}}\lambda f^2\left(1 + \frac{1}{4}\ln 3 - \ln\frac{f^2}{2}\right)\right], \tag{4.32}$$

where $\bar{n} = 2\pi n/\gamma$ and n is an integer. Though effect of the deficit angle appears in the ansatz of the global vortex (4.31), topological charge ν of it is an integer n, i.e., $\nu \equiv \oint_{\partial C_\gamma} d\vec{l}\cdot(\phi_1\vec{\partial}\phi_2 - \phi_2\vec{\partial}\phi_1)/2\pi\phi^2 = n$.

To let the configuration nonsingular, we have a boundary conditions at the origin, $f(r=0) = 0$, and, to make energy density vanish at spatial infinity, the scalar amplitude should approach to the broken vacuum at asymptotic region, $f(r=\infty) = v$. Now our task is to find a vortex string configuration interpolating these two boundary conditions and satisfying the equation of motion

$$\frac{d^2 f}{dr^2} = -\frac{1}{r}\frac{df}{dr} + \frac{\bar{n}^2}{r^2}f - \frac{d}{df}(-V_{\rm eff}), \tag{4.33}$$

where $V_{\rm eff}(f)$ is given in Eq. (3.23). Series expansion of $f(r)$ near the origin is

$$f(r) \simeq f_0 r^{\bar{n}} + \frac{5\lambda f_0^3\bar{n}}{576\pi^2(\bar{n}+1)(2\bar{n}+1)}r^{3\bar{n}+2}\ln r + \cdots, \tag{4.34}$$

where f_0 is a constant determined by the proper behavior of the fields at the asymptotic region. Though the configuration seems not to be regular at the origin due to the singularity at the vertex of the cone, Eq. (4.34) tells us that there is no δ-function-like energy addition at the origin at all. The behavior at large r is obtained precisely by analyzing the equation of motion (4.33)

$$f(r) \simeq v - \frac{144\pi^2\bar{n}^2}{5\lambda v r^2} - \frac{10368\pi^4\bar{n}^2(3\bar{n}^2+8)}{25\lambda^2 v^3 r^4} + \cdots . \tag{4.35}$$

Since we cannot have an exact solution, a numerical solution is shown in Fig. 4.[5] Profile of the energy density shows a long range tail in addition to a ring shape at the vortex core as expected. Existence of such long range term can easily be read from the energy expression (4.32) : The second term proportional to $\bar{n}^2$ is $\mathcal{O}(1/r^2)$ at large distance from the global cosmic string core so that the energy per unit length along the z-axis is logarithmically divergent as ordinary global strings did.

Now discussion on the stability of the above global string is in order under some simplified situations. Suppose that a phase transition occurred in the very early universe, *e.g.*, at the GUT scale (10^{16} GeV), and topologically-stable local cosmic strings have been produced via spontaneous symmetry breaking. If there does not happen subsequent phase transition up to the EW scale (1 TeV) like desert in the standard model and we neglect complicated dynamics of cosmic strings [1], the high-energy cosmic string wick may survive even throughout the EW phase transition because released latent heat may not be enough to melt the extremely-massive topological defects. Specifically, 1 TeV seems too small compared to 10^{16} GeV. So form of the remnant near each high-energy cosmic string is likely to be observed as a line of the deficit angle of the background spacetime or Aharonov-Bohm effect of a *magnetic* flux tube. This may justify the usage of our calculation of the effective potential in the section 2. Since the background planar space C_γ orthogonal to the string direction has a circle as its spatial boundary $\partial C_\gamma = S^1$, the low-energy global strings we obtained are also topologically stable despite its logarithmically-divergent energy.

An intriguing issue in this stage may be to answer the question whether or not a straight global string favors its generation site along the cosmic string wick. Here we deal with energy difference between two possible configurations without considering the huge mass of the background string wire, which is always same irrespective of existence of the low-energy strings so that it does not contribute to the energy difference. Suppose that a global string of unit topological charge ($n = 1$) is produced along the cosmic string

[5] Existence and uniqueness of the static global vortex solution can easily be proved. Once the following identifications, $f \to x$, $r \to t$ and $V_{\rm eff}(f) \to -V(x)$, are made, Eq. (4.33) is analogous to a Newtonian equation of a unit-mass particle in 1-dimensional motion, of which applied forces are time-dependent friction, time-dependent repulsion, and conservative force in order in Eq. (4.33). The motion corresponding to the vortex solution is that starting from the valley of the potential ($x = 0$) at $t = 0$ and arriving at the hilltop ($x = v$) after infinite time t elapsed.

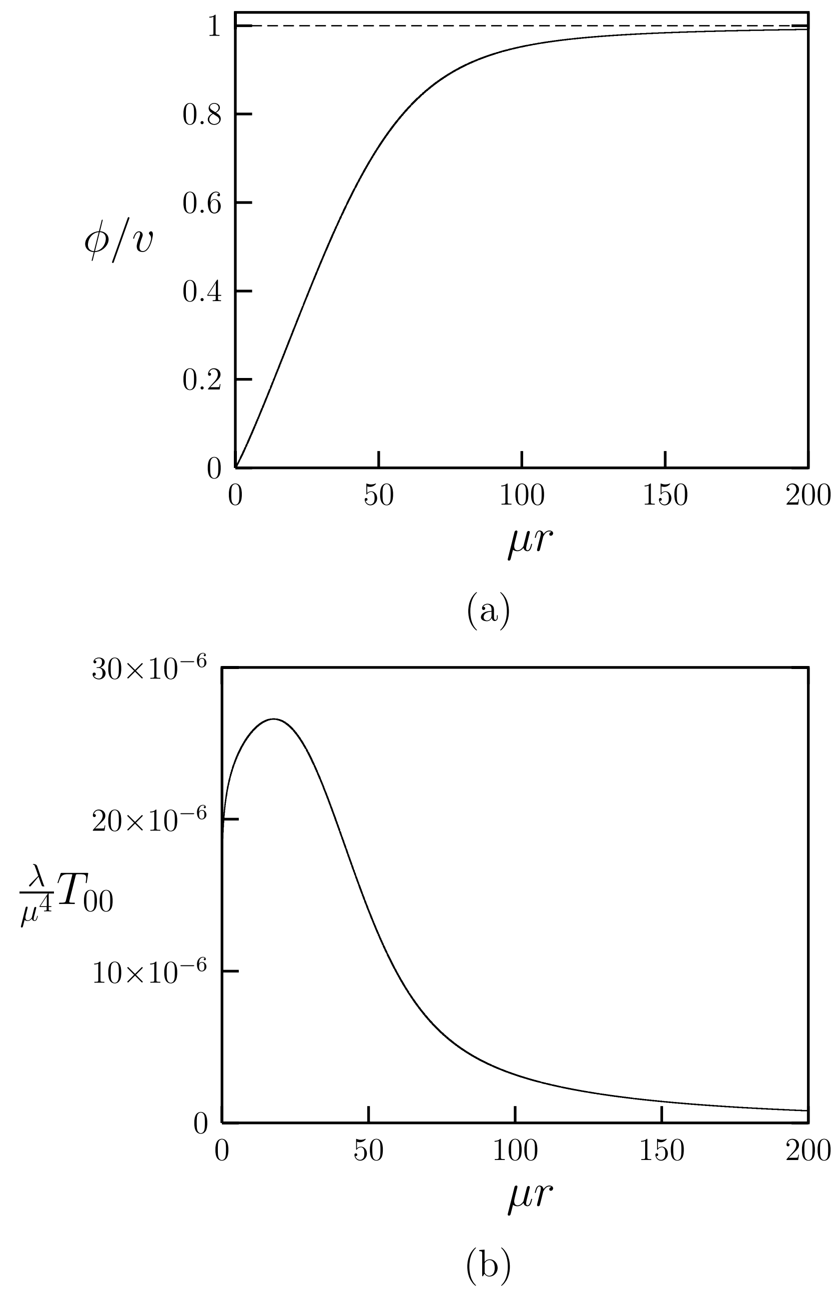

Figure 1: Profile of a global vortex when $\bar{n} = 1.1$ and $\lambda = 20$: (a) scalar amplitude ϕ, (b) energy density T_{00}.

wick. Its core size is roughly estimated from the first and second terms of Eq. (4.35), i.e., $\sqrt{\lambda}v\, r_{\rm core} \sim (1+4GM_0^2)12\pi/\sqrt{5}$. The core energy per unit length along the z-axis is computed by an integration of Eq. (4.32) from $r = 0$ to $r = r_{\rm core}$ after inserting $f(r) = 0$ into the integrands : $E_z^{\rm core} \sim \frac{\mu^2}{\lambda}\gamma \int_0^{r_{\rm core}} drr\, V_{\rm eff}(f = 0) = (1+4GM_0^2)\pi v^2/16$. The logarithmically divergent energy is obtained by another integration of Eq. (4.32) from $r = r_{\rm core}$ to an infra-red cutoff R after substituting the vacuum value $f(r) = v$: $E_z^R \sim \frac{\mu^2}{\lambda}\gamma \int_{r_{\rm core}}^R drr\, (\bar{n}^2/2r^2)|_{n=1} = (1+4GM_0^2)\pi v^2 \ln(R/r_{\rm core})$. Therefore, both the core radius and the energy per unit length along the wick are $(1+4GM_0^2)$ times those generated outside the singularity. Since $4GM_0^2 \sim 10^{-6}$ at the GUT scale, energy difference can be negligible except for the cosmic strings of Planck scale, $4GM_0^2 \sim 1$. When a global string is generated, the boundary conditions of the strings coincide with those of the vacuum (3.29) at both the origin and spatial infinity. So generation of a global string is nothing but a process that the extremely-thin wire starts to swell naturally to the extended core of radius $r_{\rm core}$ carrying a topological charge n. The energy supplied for such process may be almost the same as the energy of a global string given above. However, in order to produce a global string at the homogeneous broken vacuum, the process is rather complicated [1], some more energy cost may be needed to excite a defect in addition to that of a global string. In the context of symmetry, the former preserves the cylindrical symmetry but the latter does not. These suggest that an infinite straight global string along the supermassive thin cosmic string wick is a favorable configuration, which can naturally be generated throughout a low-energy phase transition.

5 Concluding remarks

We have analyzed vacuum structure and global string solutions in the one-loop effective action of self-interacting O(2) scalar fields of which background space has conical singularities. In the one-loop effective potential, a singular part proportional to 2-dimensional δ-function emerges in addition to well-known regular part of it. The configuration of minimum energy is an inhomogeneous vacuum in which the homogeneous broken vacuum dominates almost everywhere but the vertex points of conical singularities have symmetric vacuum. If the region of symmetric vacuum swells and then is connected smoothly to broken vacuum due to a topological winding, it becomes a hybrid of global string of low energy and local cosmic string wick of high energy.

Since our computation seems not sufficient for supporting all the steps of the procedure of consideration, refined calculation is needed in future. What we obtained in this paper can be extended to other similar cases, *e.g.*, local strings of gauge theories with a local cosmic string wick, global strings from dynamical symmetry breaking, monopoles with a supermassive global monopole seed. The obtained topological defects of candle shape with a wick should be applied for understanding various cosmological questions as the others have been examined.

Acknowledgments

We would like to thank Chanju Kim for valuable comments and discussions. Y.K. wishes to acknowledge informative discussions with T. Vachaspati, H.-C. Kim, and B. K. Lee at the early stage of this work. This work was the result of research activities (Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARCSEC)) supported by Korea Science & Engineering Foundation. O-K. Kwon is supported by the KOSEF fellowship.

References

[1] T. W. B. Kibble, J. Phys. A9 (1976) 1387;
For reviews see A. Vilenkin, E. P. S. Shellard, *Cosmic Strings and Other Topological Defects*, (Cambridge University Press, 1994);
M.B. Hindmarsh, T.W. Kibble, Rep. Prog. Phys. 58 (1995) 477.

[2] A. Vilenkin, A. E. Everett, Phys. Rev. Lett. 48 (1982) 1867.

[3] R. L. Davis, Phys. Rev. D32 (1985) 3172;
A. Vilenkin, T. Vachaspati, Phys. Rev. D35 (1987) 1138.

[4] S. L. Larson, W. A. Hiscock, Phys. Rev. D56 (1997) 3242 [gr-qc/9704028].

[5] M. Yamaguchi, J. Yokoyama, Preprint hep-ph/0205308.

[6] M. M. Forbes, A. R. Zhitnitsky, Phys. Rev. D65 (2002) 085009 [hep-ph/0109173].

[7] R. Gregory, Phys. Lett. B215 (1988) 663;
A. G. Cohen, D. B. Kaplan, *ibid.* 215 (1988) 67;
G. W. Gibbons, M. E. Ortiz, F. Ruiz Ruiz, Phys. Rev. D39 (1989) 1546.

[8] N. Kim, Y. Kim, K. Kimm, Phys. Rev. D56 (1997) 8029 [gr-qc/9707056]; Class. Quantum Grav. 15 (1998) 1513 [gr-qc/9707011].

[9] A. G. Cohen, D. B. Kaplan, Phys. Lett. B470 (1999) 52 [hep-th/9910132];
R. Gregory, Phys. Rev. Lett. 84 (2000) 2564 [hep-th/9911015];
I. Olasagasti, A. Vilenkin, Phys. Rev. D62 (2000) 044014 [hep-th/0003300];
S.-H. Moon, S.-J. Rey, Y. Kim, Nucl. Phys. B602 (2001) 467 [hep-th/0012165];
S.-H. Moon, Nucl. Phys. B624 (2002) 327 [hep-th/0112085];
R. Gregory, C. Santos, Prerint hep-th/0208037.

[10] E. Witten, Nucl. Phys. B249 (1985) 557.

[11] A. Vilenkin, Phys. Rev. D23 (1981) 852;
J. R. Gott III, Astrophys. J. 288 (1985) 422.

[12] J. S. Dowker, R. Critchley, Phys. Rev. D13 (1976) 224;
S. W. Hawking, Commun. Math. Phys. 55 (1977) 133;
For a review see N. D. Birrell, P. C. W. Davies, *Quantum Fields in Curved Space*, (Cambridge University Press, New York, 1982).

[13] J. S. Dowker, J. Phys. A10 (1977) 115;
G. Cognola, K. Kirsten, L. Vanzo, Phys. Rev. D49 (1994) 1029;
D. V. Fursaev, G. Miele, Nucl. Phys. B484 (1997) 697.

[14] R. Jackiw, pp35-53 in *Diverse Topics in Theoretical and Mathematical Physics*, (World Scientific, Singapore, 1995).